

Comment

Comment on ‘Do we have a consistent non-adiabatic quantum-classical mechanics?’

VLADIMIR V. KISIL^(a), KISILV@MATHS.LEEDS.AC.UK

School of Mathematics, University of Leeds, Leeds LS2 9JT, UK. Web: <http://maths.leeds.ac.uk/~kisilv/>

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Introduction. – This is a comment on the paper [1], which evaluates the quantum-classical (QC) bracket:

$$[K_1, K_2]_{qc} = \frac{1}{i\hbar}[K_1, K_2] + \frac{1}{2}(\{K_1, K_2\} - \{K_2, K_1\}) - i\partial_{h_2}[K_1, K_2]|_{h_2=0}, \quad (1)$$

introduced in paper [4, (26)]. The authors in [1] claimed that the QC bracket (1) exhibits:

- an artificial coupling property (i.e., coupling between the subsystems in the absence of an interaction);
- a genuinely classical nature (i.e., the apparent mixed quantum classical form reduces to a purely classical form for both subsystems).

The assessment in [1] oversaw the following points:

1. QC bracket (1) is the image of the universal bracket [4, (22)]:

$$\{[k_1, k_2]\} = (k_1 * k_2 - k_2 * k_1)(\mathcal{A}_1 + \mathcal{A}_2), \quad (2)$$

under QC representation [4, (20)]. The universal bracket consists of convolution commutator and antiderivative operators [4, (12)]. QC bracket requires consideration of the first jet space [4]: the bracket is determined not only by their values of observables at $h_2 = 0$ but also by the values of their first derivative with respect to h_2 at zero (see the last term in (1)).

2. The derivation QC bracket (1) is independent of p-mechanisation procedure introduced in [4, (23)]:

$$q_j \mapsto Q_j = \delta'_{x_j}(g_1; g_2), \quad (3)$$

$$p_1 \mapsto P_j = \chi'_{s_k}(s_1 + s_2) * \delta'_{y_j}(g_1; g_2), \quad (4)$$

where $j = 1, 2$ and $k = 3 - j$.

Here *p-mechanisation* [3, § 3.3], as an analog of quantisation, is a prescription how to build p-mechanical observables out of classical ones. It may not be very explicit in [4], but the deduction of the bracket (1) is compatible with different choices of p-mechanisation, however the value of the bracket will be different, see Exs. 3 and 4 below. To illustrate this in the present comment we use p-mechanisation given by the Weyl (symmetric) calculus based on the following correspondence, cf. [4, (23)], [1, (19)] and (3)–(4):

$$q_j \mapsto Q_j = \delta'_{x_j}(g_1; g_2), \quad p_j \mapsto P_j = \delta'_{y_j}(g_1; g_2), \quad (5)$$

Then the quantum-quantum image of the universal bracket (2) of the respective coordinate and momentum observables is:

$$[Q_j, P_j]_{qq} = \frac{h_1 + h_2}{h_k} I, \quad k = 3 - j. \quad (6)$$

Now we review the above two claims from the paper [1].

Artificial coupling property. – There is the following claim in [1, 3001-p3]: “It must be underlined that eq. (16) describes an artificial interaction even if the two systems are not coupled by the Hamiltonian.” This coupling property is attributed to the fact that quantum-quantum bracket in [4, (25)] and [1, (16)] always contains both Planck’s constants h_1 and h_2 , which are generated by the presence of both antiderivative operators in the definition of universal bracket (2).

Example 1. In order to exam the claim let us consider an uncoupled Hamiltonian $H(q_1, p_1, q_2, p_2) = H_1(q_1, p_1) + H_2(q_2, p_2)$. The p-mechanisation (as well as quantisation) is a linear map [3, § 3.3], thus this uncoupled structure will be preserved. Let \hat{B} be an observable depending only from \hat{X}_2 and \hat{D}_2 , thus it will commute with H_1 . Therefore

^(a) On leave from Odessa University

the commutator of B and H will be the same as B and H_2 . The QC bracket is an image under a representation of the usual commutator, thus the universal bracket (2) of B and H will be the same as B and H_2 . Consequently the \hat{H}_1 will not affect the dynamics of such an observable \hat{B} .

Therefore there is no coupling in the following meaning: arbitrary change of the Hamiltonian H_1 of the first subsystem will not affect dynamics of any observable build from coordinates and momenta of the second system only.

Genuinely classical nature. – The paper [1, 3001-p3] said “In ref. [8] it was suggested that the dynamical equation (16), in the limit $h_1 = h$ and $h_2 \rightarrow 0$, yields a QC dynamics.” However the derivation in [4] of the QC bracket intentionally avoids any kind of semiclassical limits due to its potential danger, see such an attempt in [2] and Example 2 below. The actual method evaluates the image of the universal bracket (2) under the QC representation [4, (20)] of the group \mathbb{D}^m .

The paper [1, 3001-p4] “corrected” the original derivation of QC bracket replacing the initial set of Planck constants h_1 and to h_2 by the new one h_{eff} defined by the expression:

$$\frac{1}{h_{\text{eff}}} = \frac{1}{h_1} + \frac{1}{h_2}. \quad (7)$$

However this transformation is singular for $h_1 h_2 = 0$ and needs special clarifications how to proceed for such values.

Example 2. Let us consider the transformation $U_h : f(x, y) \mapsto f(hx, \frac{1}{h}y)$, which is a unitary operator $L_2(\mathbb{R}^2) \rightarrow L_2(\mathbb{R}^2)$ for any $h > 0$. However this does not allow us “to take the limit $h \rightarrow 0$ ” through the straightforward substitution $h = 0$.

Furthermore the paper [1, 3001-p3] claims that “we have shown that the equation of motion (16) does not lead to a non-trivial QC limit”. However, this is caused by p-mechanisation (3)–(4), cf. the next two examples.

Example 3. Let B_1 and B_2 are squares of coordinate Q and momentum P observables (of the quantum subsystem) respectively. Under p-mechanisation (3)–(4) they are represented by squares of corresponding convolutions. Then the commutator (first term of bracket (1)) of their QC representations is zero, the second term in (1)) vanishes since no classical observables present, and the third term in (1)) is equal to QC image of the observable $4QP$. Thus the total bracket is indeed the same as the Poisson bracket for those observables.

Let us examine the above claim for the p-mechanisation (5) and assume that two p-mechanical observables B_1 and B_2 , that is two convolutions on the group \mathbb{D}^n [4, p. 876], for any fixed g_1 are multiples of the delta function in g_2 , e.g. as in Ex. 3. Under the QC representation $\rho_{(h;q,p)}$ [4, (20)] those observables become operators $\rho_{(h;q,p)}(B_1)$ and $\rho_{(h;q,p)}(B_2)$ on the state space for the quantum subsystem without any dependence from classical coordinates p, q and the respective Planck constant h_2 . Correspondingly the second and the third terms of

the bracket (1) vanish and this bracket is equal to the (quantum) commutator $\frac{1}{i\hbar}[\rho_{(h;q,p)}(B_1), \rho_{(h;q,p)}(B_2)]$.

Therefore if we admit the claim [1, 3001-p3] that QC bracket (1) always coincides with the purely classic Poisson bracket, then we have to accept that any quantum commutator is always equal to the Poisson bracket.

Example 4. Under p-mechanisation (5) the squares of momentum and coordinates from Ex. 3 are represented by convolutions with kernels $\delta''_{x_1 x_1}(g_1; g_2)$ and $\delta''_{y_1 y_1}(g_1; g_2)$. Their commutator on the group \mathbb{D}^1 is $4\delta'''_{x_1 y_1 s_1} + 2\delta''_{s_1 s_1}$. Thus the universal bracket (2) is

$$\{[B_1, B_2]\} = 4\delta''_{x_1 y_1} + 2\delta'_{s_1} + (4\delta'''_{x_1 y_1 s_1} + 2\delta''_{s_1 s_1})A_2. \quad (8)$$

In the QC representation of \mathbb{D}^1 the last term of the sum vanishes and two first terms produce $4QP + 2i\hbar I$. This is the quantum commutator of Q^2 and P^2 times $\frac{1}{i\hbar}$. There is no unitary representation to get rid of the purely imaginary term $2i\hbar I$ in order to reduce the QC bracket of B_1 and B_2 to the value $4QP$ of their Poisson bracket.

Conclusion. – In this paper we demonstrated that the QC bracket (1) does not possess itself two properties of “artificial coupling” and “genuinely classical nature” as claimed in [1]. Unfortunately the claims [1] were uncritically translated by some other authors, see [5, 6].

We showed that for a decoupled Hamiltonian the dynamics of observables localised in one subsystem is unaffected by the Hamiltonian of the other subsystem. The “classical nature” described in [1] is rooted in the p-mechanisation used in [4] and does not appear with other choice of p-mechanical observables.

The main conclusion of the commented paper [1] is: “We suggest that a different Ansatz for the equations of motion, could indeed produce non-trivial QC equations”. This comment is aimed to clarify possible directions for such a search.

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